

Calculus Bowl: Expert Edition

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<https://intuitiveexplanations.com/math/calculus-bowl-expert-edition/>

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Question 1

For what strictly positive number x is x^x the smallest?

- A. $1/e$
- B. $\ln 2$
- C. 1
- D. e
- E. There is no such x

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Question 2

Find the value of $\sum_{k=0}^{\infty} e^{-\pi k} \cos(-\pi k)$.

A. $\frac{1}{1 + e^{\pi}}$

B. $\frac{1}{1 - e^{\pi}}$

C. $\frac{e^{\pi}}{1 + e^{\pi}}$

D. $\frac{e^{(\pi^2)}}{1 - e^{(\pi^2)}}$

E. $\frac{e^{(\pi^2)}}{e^{(\pi^2)} - 1}$

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E. $\frac{e^{(\pi^2)}}{e^{(\pi^2)} - 1}$

Question 3

What is the area in Quadrant IV above the curve $y = \ln x$?

- A. 1
- B. $\ln 2$
- C. $e - 1$
- D. e
- E. ∞

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Question 4

Find the slope of the tangent line to

$$y = \frac{9x^3 - 27x^2 + 5x - 6}{55x^4 + 11x^3 - 7x^2 + 6x + 3} \text{ at } x = 0.$$

- A. 1
- B. 2
- C. 3
- D. 4
- E. 6

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A. 1

B. 2

C. 3

D. 4

E. 6

Question 5

Find $\lim_{x \rightarrow \infty} \sin(\arctan x)$.

- A. -1
- B. 0
- C. $1/2$
- D. 1
- E. The limit does not exist

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- A. -1
- B. 0
- C. $1/2$
- D. 1
- E. The limit does not exist

Question 6

Let $f(x) = \begin{cases} mx + b & x \leq 1 \\ x^2 & x > 1 \end{cases}$. If f is everywhere differentiable, what are m and b ?

- A. $m = -2, b = -1$
- B. $m = 3, b = -2$
- C. $m = 2, b = -1$
- D. $m = 1, b = 0$
- E. $m = 2, b = -2$

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Question 7

If $x = y^2$, $y = z^3$, and $z = w^4$, then what is $\frac{dx}{dy} + \frac{dx}{dz} + \frac{dx}{dw}$?

- A. $2w^{12} + 6w^{20} + 24w^{23}$
- B. $2w^8 + 3w^6 + 4w^3$
- C. $2w^{12} + 3w^8 + 4w^3$
- D. $2w + 3w^2 + 4w^3$
- E. $2w + 6w^5 + 24w^{23}$

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Question 8

Suppose that t is measured in meters and $v(t)$ is measured in seconds. What are the units of $\int v'(t) dt$?

- A. s
- B. m
- C. m/s^2
- D. m/s
- E. s/m

Question 8

Suppose that t is measured in meters and $v(t)$ is measured in seconds. What are the units of $\int v'(t) dt$?

- A. s
- B. m
- C. m/s²
- D. m/s
- E. s/m

Question 9

Let f be a continuous function such that $f(x) + f(1 - x) \neq 0$ for all

x . Evaluate $\int_0^1 \frac{f(x)}{f(x) + f(1 - x)} dx$.

- A. 0
- B. Cannot be determined without more information
- C. $1/2$
- D. 1
- E. 2

Question 9

Let f be a continuous function such that $f(x) + f(1 - x) \neq 0$ for all

x . Evaluate $\int_0^1 \frac{f(x)}{f(x) + f(1 - x)} dx$.

- A. 0
- B. Cannot be determined without more information
- C. $1/2$
- D. 1
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Question 10

Suppose that $\int_{-3}^4 f(x) dx = 3$, $\int_{-1}^7 f(x) dx = 7$, and $\int_{-3}^7 f(x) dx = 5$.

What is $\int_{-1}^4 f(x) dx$?

- A. 5
- B. 8
- C. 2
- D. 10
- E. -5

Question 10

Suppose that $\int_{-3}^4 f(x) dx = 3$, $\int_{-1}^7 f(x) dx = 7$, and $\int_{-3}^7 f(x) dx = 5$.

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Question 11

Find the value of $\sum_{n=0}^{\infty} \frac{\sin(\frac{n\pi}{2})}{n!}$.

- A. $\cos 1$
- B. $\ln 2$
- C. $\sin 1$
- D. 1
- E. e

Question 11

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B. $\ln 2$

C. $\sin 1$

D. 1

E. e

Question 12

If $x^2 + y^2 = 1$, then what is $\frac{d^2y}{dx^2}$?

A. $\frac{y^3}{x^2 - y^2}$

B. $-\frac{1}{y^3}$

C. $-\frac{x}{y}$

D. $-\frac{3x}{y^5}$

E. $\frac{y^2 - x^2}{y^3}$

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E. $\frac{y^2 - x^2}{y^3}$

Question 13

Which of the following is **not** a correct solution to the problem

$$\int 2 \tan x \sec^2 x \, dx?$$

A. $\frac{2}{1 + \cos 2x}$

B. $\sec^2 x$

C. $\sqrt{\frac{1 - \cos x}{1 + \cos x}}$

D. $\frac{4}{(e^{ix} + e^{-ix})^2}$

E. $\frac{1 - \cos 2x}{1 + \cos 2x}$

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D. $\frac{4}{(e^{ix} + e^{-ix})^2}$

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Question 14

Rolle's theorem is a special case of which of the following theorems?

- A. Extreme value theorem
- B. Mean value theorem
- C. Fundamental theorem of calculus
- D. Intermediate value theorem
- E. Mean value theorem for integrals

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- E. Mean value theorem for integrals

Question 15

Suppose that the position of a particle is given by the equation $x = \sin(\pi t)$. Find its velocity when $t = 1/6$, given that x is time and t is position.

- A. $\frac{\sqrt{3}\pi}{2}$
- B. $\frac{2}{\sqrt{3}\pi}$
- C. $\frac{4}{3\sqrt{3}\pi}$
- D. $\frac{6}{\sqrt{35}\pi}$
- E. 0

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- E. 0

Question 16

What is the behavior of the expression $\lim_{m \rightarrow \infty} \lim_{n \rightarrow \infty} \cos^{2n}(m!\pi x)$?
Assume m and n are integers.

- A. It is always zero
- B. It is always one
- C. It does not exist
- D. It is zero if x is irrational and one if x is rational
- E. It is zero if x is not an integer and one if x is an integer

Question 16

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Question 17

Identify the differential equations whose solutions would be underestimated by Euler's method.

I. $\frac{dx}{dt} = 1$

II. $\frac{dx}{dt} = t$

III. $\frac{dx}{dt} = x$

A. II only

B. III only

C. I and II only

D. II and III only

E. I, II, and III

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D. II and III only

E. I, II, and III

Question 18

Find $\int_{\pi}^{\pi^2} \frac{\pi^2 \ln(\pi^\pi)}{\theta(\pi + \pi^2)} d\theta$.

A. $\frac{2\pi(\ln \pi)^2}{1 + \pi}$

B. $\frac{\pi^2 \ln \pi}{1 + \pi}$

C. $\frac{2\pi^2(\ln \pi)^2}{1 + \pi}$

D. $\frac{\pi^2(\ln \pi)^2}{1 + \pi}$

E. $\frac{\pi^3(\pi - 1) \ln \pi}{\pi + 1}$

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C. $\frac{2\pi^2(\ln \pi)^2}{1 + \pi}$

D. $\frac{\pi^2(\ln \pi)^2}{1 + \pi}$

E. $\frac{\pi^3(\pi - 1) \ln \pi}{\pi + 1}$

Question 19

Determine the surface area of the solid generated by revolving the curve $y = \sqrt{4 - x^2}$ about the x -axis.

- A. 2π
- B. 4π
- C. $\frac{16}{3}\pi$
- D. 8π
- E. 16π

Question 19

Determine the surface area of the solid generated by revolving the curve $y = \sqrt{4 - x^2}$ about the x -axis.

- A. 2π
- B. 4π
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- E. 16π

Question 20

Which of the following integrals, when evaluated, gives the area of one of the regions bounded by the curves $y = \cos x$ and $y = \sin x$?

A. $\int_{\pi/4}^{5\pi/4} \cos x - \sin x \, dx$

B. $\int_0^{\pi/4} \cos x - \sin x \, dx$

C. $\int_{\pi/4}^{-3\pi/4} \sin x - \cos x \, dx$

D. $\int_0^{\pi} |\cos x| - |\sin x| \, dx$

E. $\int_0^{\pi} \cos x - \sin x \, dx$

Question 20

Which of the following integrals, when evaluated, gives the area of one of the regions bounded by the curves $y = \cos x$ and $y = \sin x$?

A. $\int_{\pi/4}^{5\pi/4} \cos x - \sin x \, dx$

B. $\int_0^{\pi/4} \cos x - \sin x \, dx$

C. $\int_{\pi/4}^{-3\pi/4} \sin x - \cos x \, dx$

D. $\int_0^{\pi} |\cos x| - |\sin x| \, dx$

E. $\int_0^{\pi} \cos x - \sin x \, dx$

Question 21

What is the k th derivative of x^n ?

A. $k!x^{n-k}$

B. $\frac{n!}{(n-k-1)!}x^{n-k}$

C. $\frac{n!}{(n-k)!}x^{n-k}$

D. $n!x^{n-k}$

E. $(n-k)!x^{n-k}$

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C. $\frac{n!}{(n-k)!}x^{n-k}$

D. $n!x^{n-k}$

E. $(n-k)!x^{n-k}$

Question 22

What is the average value of the function $f(x) = \sqrt{1 - x^2}$ over its domain?

- A. $\pi/4$
- B. $\pi/6$
- C. $3/4$
- D. $\sqrt{3}/2$
- E. $5/8$

Question 22

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- B. $\pi/6$
- C. $3/4$
- D. $\sqrt{3}/2$
- E. $5/8$

Question 23

Evaluate $\lim_{x \rightarrow \infty} \frac{\sum_{n=3}^9 nx^n}{\sqrt[3]{\sum_{n=9}^{27} \frac{x^n}{n}}}.$

- A. 0
- B. 3
- C. 9
- D. 27
- E. ∞

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- A. 0
- B. 3
- C. 9
- D. 27
- E. ∞

Question 24

Evaluate $\frac{d}{dx} [\ln \ln \ln \ln x]$ at $x = e^{(e^e)}$.

- A. e^{-e-1}
- B. e^{e^e+e+1}
- C. e^{-e^e-e-1}
- D. $e^{-e^{-e-1}-e}$
- E. e^{-e^2-e-1}

Question 24

Evaluate $\frac{d}{dx} [\ln \ln \ln \ln x]$ at $x = e^{(e^e)}$.

- A. e^{-e-1}
- B. e^{e^e+e+1}
- C. e^{-e^e-e-1}
- D. $e^{-e^{-e-1}-e}$
- E. e^{-e^2-e-1}

Question 25

Find the value of $\lim_{x \rightarrow 0} \frac{\ln(1-x) - \sin x}{1 - \cos^2 x}$.

- A. The limit does not exist!
- B. ∞
- C. $1/2$
- D. $-\infty$
- E. 0

Question 25

Find the value of $\lim_{x \rightarrow 0} \frac{\ln(1-x) - \sin x}{1 - \cos^2 x}$.

A. The limit does not exist!

B. ∞

C. $1/2$

D. $-\infty$

E. 0

Question 26

Taken collectively, how many different intervals of convergence do the following series have?

$$\sum_{n=1}^{\infty} r^n$$

$$\sum_{n=1}^{\infty} n^{-r}$$

$$\sum_{n=1}^{\infty} r^{-n}$$

$$\sum_{n=1}^{\infty} n^r$$

$$\sum_{n=1}^{\infty} (-r)^n$$

$$\sum_{n=1}^{\infty} (-r)^{-n}$$

- A. 1
- B. 2
- C. 3
- D. 4
- E. 6

Question 26

Taken collectively, how many different intervals of convergence do the following series have?

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- A. 1
- B. 2
- C. 3
- D. 4
- E. 6

Question 27

How many points of inflection can a polynomial of degree 7 have, at most?

- A. 3
- B. 5
- C. 6
- D. 7
- E. 8

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- B. 5
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- E. 8

Question 28

Evaluate $\frac{d}{dx} [(\ln x)^{\ln x}]$.

A. $(\ln x)^{\ln x} \left(\frac{\ln x + \ln \ln x}{x} \right)$

B. $(\ln x)^{\ln x} \left(\frac{1 + \ln \ln x}{x} \right)$

C. $(\ln x)^{\ln x} \left(\frac{1 + \ln x}{\ln \ln x} \right)$

D. $(\ln x)^{\ln x} \left(1 + \frac{\ln \ln x}{x} \right)$

E. $(\ln x)^{\ln x} \left(\frac{1 + \ln \ln x}{\ln x} \right)$

Question 28

Evaluate $\frac{d}{dx} [(\ln x)^{\ln x}]$.

A. $(\ln x)^{\ln x} \left(\frac{\ln x + \ln \ln x}{x} \right)$

B. $(\ln x)^{\ln x} \left(\frac{1 + \ln \ln x}{x} \right)$

C. $(\ln x)^{\ln x} \left(\frac{1 + \ln x}{\ln \ln x} \right)$

D. $(\ln x)^{\ln x} \left(1 + \frac{\ln \ln x}{x} \right)$

E. $(\ln x)^{\ln x} \left(\frac{1 + \ln \ln x}{\ln x} \right)$

Question 29

Evaluate $\int_{-\infty}^0 x^5 e^x dx$.

- A. -120
- B. -24
- C. -5
- D. -1
- E. The integral cannot be expressed in closed form

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Question 30

The power rule states that $\frac{d}{dx}[x^n] = nx^{n-1}$. In which of the following situations does the power rule **not** hold?

- I. $x = 0$ and $n < 1$
 - II. $x = 0$ and $n = 1$
 - III. $x < 0$ and n is irrational
- A. None
 - B. I only
 - C. II only
 - D. I and II only
 - E. I, II, and III

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- I. $x = 0$ and $n < 1$
 - II. $x = 0$ and $n = 1$
 - III. $x < 0$ and n is irrational
- A. **None**
 - B. **I only**
 - C. **II only**
 - D. **I and II only**
 - E. **I, II, and III**

Question 31

Compute $\int_0^{\infty} (e^{-x})^2 dx$.

- A. $1/2$
- B. $\sqrt{\pi}/2$
- C. 1
- D. $\sqrt{\pi}$
- E. The integral does not converge

Question 31

Compute $\int_0^{\infty} (e^{-x})^2 dx$.

- A. $1/2$
- B. $\sqrt{\pi}/2$
- C. 1
- D. $\sqrt{\pi}$
- E. The integral does not converge

Question 32

If the arc length of the curve given by $x(t) = \cos \sqrt{t}$ and $y(t) = \sin \sqrt{t}$ from $t = 0$ to $t = T$ is 6π , then what is T ?

- A. $\sqrt{3}$
- B. $\sqrt{6}$
- C. 3
- D. 6
- E. 9

Question 32

If the arc length of the curve given by $x(t) = \cos \sqrt{t}$ and $y(t) = \sin \sqrt{t}$ from $t = 0$ to $t = T$ is 6π , then what is T ?

A. $\sqrt{3}$

B. $\sqrt{6}$

C. 3

D. 6

E. 9

Question 33

Evaluate $\frac{d}{dx} [x + x^2 + x^3 + \cdots + x^{100}]$ at $x = 1$.

- A. 99
- B. 100
- C. 5050
- D. 10100
- E. 100!

Question 33

Evaluate $\frac{d}{dx} [x + x^2 + x^3 + \cdots + x^{100}]$ at $x = 1$.

- A. 99
- B. 100
- C. 5050
- D. 10100
- E. 100!

Question 34

What is the rate of change of the length of the hypotenuse of an isosceles right triangle with respect to the length of one of its legs?

- A. 0
- B. 1
- C. $\sqrt{2}$
- D. 2
- E. $2\sqrt{2}$

Question 34

What is the rate of change of the length of the hypotenuse of an isosceles right triangle with respect to the length of one of its legs?

- A. 0
- B. 1
- C. $\sqrt{2}$
- D. 2
- E. $2\sqrt{2}$

Question 35

What is $\frac{d}{dA} [ABRACADABRA]$ when $A = 1$, $B = 2$, $C = 3$, and $D = 4$?

- A. $5R^2$
- B. $48R^2$
- C. $120R^2$
- D. $192R^2$
- E. $240R^2$

Question 35

What is $\frac{d}{dA}[ABRACADABRA]$ when $A = 1$, $B = 2$, $C = 3$, and $D = 4$?

- A. $5R^2$
- B. $48R^2$
- C. $120R^2$
- D. $192R^2$
- E. $240R^2$

Question 36

Determine the value of $\frac{d}{dx} \left[|\sin x| + |\cos x| + |\tan x| \right]$ at $x = \frac{\pi}{4}$.

- A. 1
- B. $\sqrt{2}$
- C. $1 + \frac{\sqrt{2}}{2}$
- D. 2
- E. $1 + \sqrt{2}$

Question 36

Determine the value of $\frac{d}{dx} \left[|\sin x| + |\cos x| + |\tan x| \right]$ at $x = \frac{\pi}{4}$.

- A. 1
- B. $\sqrt{2}$
- C. $1 + \frac{\sqrt{2}}{2}$
- D. 2
- E. $1 + \sqrt{2}$

Question 37

Compute $\frac{d}{dx} \left[\frac{x}{\ln x} \right]$.

A. $\frac{1}{\ln x} + \frac{1}{(\ln x)^2}$

B. $\frac{1}{\ln x} - \frac{1}{(\ln x)^2}$

C. $\frac{1}{x^2} - \frac{\ln x}{x^2}$

D. $\frac{1}{(\ln x)^2} - \frac{1}{\ln x}$

E. $\frac{\ln x - 1}{\ln(x^2)}$

Question 37

Compute $\frac{d}{dx} \left[\frac{x}{\ln x} \right]$.

A. $\frac{1}{\ln x} + \frac{1}{(\ln x)^2}$

B. $\frac{1}{\ln x} - \frac{1}{(\ln x)^2}$

C. $\frac{1}{x^2} - \frac{\ln x}{x^2}$

D. $\frac{1}{(\ln x)^2} - \frac{1}{\ln x}$

E. $\frac{\ln x - 1}{\ln(x^2)}$

Question 38

Suppose that the radius of a sphere is 1 inch, and its volume is increasing at a rate of 1 cubic inch per minute. How fast is its surface area increasing?

- A. $1 \text{ in}^2/\text{min}$
- B. $2 \text{ in}^2/\text{min}$
- C. $2\pi \text{ in}^2/\text{min}$
- D. $4\pi \text{ in}^2/\text{min}$
- E. $8\pi \text{ in}^2/\text{min}$

Question 38

Suppose that the radius of a sphere is 1 inch, and its volume is increasing at a rate of 1 cubic inch per minute. How fast is its surface area increasing?

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- D. $4\pi \text{ in}^2/\text{min}$
- E. $8\pi \text{ in}^2/\text{min}$

Question 39

What is the average value of $\arctan x$ from $x = 0$ to $x = 1$?

- A. $\frac{\pi}{4} - \frac{1}{2}$
- B. $\frac{\pi}{4} - \ln \sqrt{2}$
- C. $\frac{1}{4} + \ln 2$
- D. $\frac{\pi}{2} - \frac{3}{4}$
- E. $\ln \left(\frac{\pi}{2} \right)$

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- B. $\frac{\pi}{4} - \ln \sqrt{2}$
- C. $\frac{1}{4} + \ln 2$
- D. $\frac{\pi}{2} - \frac{3}{4}$
- E. $\ln \left(\frac{\pi}{2} \right)$

Question 40

Find the average value of $\tan x$ for x in the interval $[0, \pi/4]$.

- A. $\frac{1}{2}$
- B. $\frac{2}{\pi} + \frac{1}{4}$
- C. $1 - \frac{2}{\pi}$
- D. $\frac{\pi}{4}$
- E. $\frac{4}{\pi} - 1$

Question 40

Find the average value of $\tan x$ for x in the interval $[0, \pi/4]$.

- A. $\frac{1}{2}$
- B. $\frac{2}{\pi} + \frac{1}{4}$
- C. $1 - \frac{2}{\pi}$
- D. $\frac{\pi}{4}$
- E. $\frac{4}{\pi} - 1$

Question 41

Find $\lim_{x \rightarrow 0} |x|^x$.

- A. 0
- B. $1/2$
- C. 1
- D. ∞
- E. The limit does not exist

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Question 42

Evaluate $\frac{d}{dx} \int_x^{2x} \frac{x}{t} dt$.

- A. $2x$
- B. $\ln x$
- C. $\ln 2$
- D. 1
- E. x

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- B. $\ln x$
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Question 43

Find $\int_{-1}^2 \frac{dx}{x}$.

- A. 0
- B. $\ln 2$
- C. 1
- D. e
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Question 44

For what value of p is the expression

$$\lim_{n \rightarrow \infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{n}}{n^p} \text{ finite and nonzero?}$$

- A. $1/2$
- B. 1
- C. $3/2$
- D. 2
- E. $5/2$

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A. $1/2$

B. 1

C. $3/2$

D. 2

E. $5/2$

Question 45

Suppose that $x^y = y^x$. Find $\frac{dy}{dx}$.

- A. $\frac{x^2 \ln y - xy}{y^2 \ln x - xy}$
- B. $\frac{xy \ln y - y^2}{xy \ln x - x^2}$
- C. $\frac{xy \ln y - x^2}{xy \ln x - y^2}$
- D. $\frac{x^2 \ln x - y^2}{y^2 \ln y - x^2}$
- E. $\frac{xy \ln x - y^2}{xy \ln y - x^2}$

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Question 46

Consider a collection of spheres with radii given by $\left\{1, \frac{1}{2^p}, \frac{1}{3^p}, \dots\right\}$. If the total volume is finite but the total surface area is infinite, then what values of p are possible?

- A. $2 \leq p < 3$
- B. $\frac{1}{3} < p \leq \frac{1}{2}$
- C. $2 < p \leq 3$
- D. $\frac{1}{3} < p \leq 1$
- E. $1 < p \leq 3$

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Question 47

Determine the value of $\sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}$.

- A. $\frac{\sqrt{2}}{5}$
- B. $\frac{1 - \sqrt{5}}{2}$
- C. $\frac{1 - \sqrt{3}}{2}$
- D. $\frac{1 + \sqrt{5}}{2}$
- E. $\frac{1 + \sqrt{3}}{2}$

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- C. $\frac{1 - \sqrt{3}}{2}$
- D. $\frac{1 + \sqrt{5}}{2}$
- E. $\frac{1 + \sqrt{3}}{2}$

Question 48

Determine $\frac{d}{dx}[(\ln x)^e]$.

A. $\frac{e}{x}(\ln x)^{e-1}$

B. $(\ln \ln x)(\ln x)^e$

C. $\frac{e}{x}$

D. $(\ln x)^e \left(1 + \frac{e}{x}\right)$

E. 1

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B. $(\ln \ln x)(\ln x)^e$

C. $\frac{e}{x}$

D. $(\ln x)^e \left(1 + \frac{e}{x}\right)$

E. 1

Question 49

Suppose that f is a function defined on an interval I . Which of the following statements **must** be true?

- I. If f is continuous on I , then f is differentiable on I .
 - II. If f is differentiable on I , then f is continuous on I .
 - III. If f is integrable on I , then f is continuous on I .
 - IV. If f is continuous on I , then f is integrable on I .
- A. II only
 - B. I and III only
 - C. II and IV only
 - D. II, III, and IV only
 - E. I, II, III, and IV

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