Calculus Bowl: Expert Edition

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https://intuitiveexplanations.com/calculus-bowl-expert-edition/

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Question 1

For what strictly positive number x is x^x the smallest?

A. $1/e$

B. $\ln 2$

C. 1

D. e

E. There is no such x

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A. $1/e$

B. $\ln 2$

C. 1

D. e

E. There is no such x

Question 2

Find the value of $\sum_{k=0}^{\infty} e^{-\pi k} \cos(-\pi k)$.

A. $\dfrac{1}{1+e^{\pi}}$

B. $\dfrac{1}{1-e^{\pi}}$

C. $\dfrac{e^{\pi}}{1+e^{\pi}}$

D. $\dfrac{e^{(\pi^2)}}{1-e^{(\pi^2)}}$

E. $\dfrac{e^{(\pi^2)}}{e^{(\pi^2)}-1}$

Question 2

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C. $\dfrac{e^{\pi}}{1+e^{\pi}}$

D. $\dfrac{e^{(\pi^2)}}{1-e^{(\pi^2)}}$

E. $\dfrac{e^{(\pi^2)}}{e^{(\pi^2)}-1}$

Question 3

What is the area in Quadrant IV above the curve $y = \ln x$?

A. 1

B. $\ln 2$

C. $e - 1$

D. e

E. ∞

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A. 1

B. $\ln 2$

C. $e - 1$

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Question 4

Find the slope of the tangent line to $y = \dfrac{9x^3 - 27x^2 + 5x - 6}{55x^4 + 11x^3 - 7x^2 + 6x + 3}$ at $x = 0$.

A. 1

B. 2

C. 3

D. 4

E. 6

Question 4

Find the slope of the tangent line to $y = \dfrac{9x^3 - 27x^2 + 5x - 6}{55x^4 + 11x^3 - 7x^2 + 6x + 3}$ at $x = 0$.

A. 1

B. 2

C. 3

D. 4

E. 6

Question 5

Find $\lim_{x \to \infty} \sin(\arctan x)$.

A. -1

B. 0

C. $1/2$

D. 1

E. The limit does not exist

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Question 6

Let $f(x) = \begin{cases} mx + b & x \leq 1 \\ x^2 & x > 1 \end{cases}$. If f is everywhere differentiable, what are m and b?

A. $m = -2,\ b = -1$

B. $m = 3,\ b = -2$

C. $m = 2,\ b = -1$

D. $m = 1,\ b = 0$

E. $m = 2,\ b = -2$

Question 6

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B. $m = 3,\ b = -2$

C. $m = 2,\ b = -1$

D. $m = 1,\ b = 0$

E. $m = 2,\ b = -2$

Question 7

If $x = y^2$, $y = z^3$, and $z = w^4$, then what is $\frac{dx}{dy} + \frac{dx}{dz} + \frac{dx}{dw}$?

A. $2w^{12} + 6w^{20} + 24w^{23}$

B. $2w^8 + 3w^6 + 4w^3$

C. $2w^{12} + 3w^8 + 4w^3$

D. $2w + 3w^2 + 4w^3$

E. $2w + 6w^5 + 24w^{23}$

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C. $2w^{12} + 3w^8 + 4w^3$

D. $2w + 3w^2 + 4w^3$

E. $2w + 6w^5 + 24w^{23}$

Question 8

Suppose that t is measured in meters and $v(t)$ is measured in seconds. What are the units of $\int v''(t)\,dt$?

A. s

B. m

C. m/s^2

D. m/s

E. s/m

Question 8

Suppose that t is measured in meters and $v(t)$ is measured in seconds. What are the units of $\int v''(t)\,dt$?

A. s

B. m

C. m/s^2

D. m/s

E. s/m

Question 9

Let f be a continuous function such that $f(x) + f(1-x) \neq 0$ for all x. Evaluate $\displaystyle\int_0^1 \frac{f(x)}{f(x)+f(1-x)}\,dx$.

A. 0

B. Cannot be determined without more information

C. 1/2

D. 1

E. 2

Question 9

Let f be a continuous function such that $f(x) + f(1-x) \neq 0$ for all x. Evaluate $\displaystyle\int_0^1 \frac{f(x)}{f(x) + f(1-x)}\, dx$.

A. 0

B. Cannot be determined without more information

C. 1/2

D. 1

E. 2

Question 10

Suppose that $\int_{-3}^{4} f(x)\,dx = 3$, $\int_{-1}^{7} f(x)\,dx = 7$, and $\int_{-3}^{7} f(x)\,dx = 5$. What is $\int_{-1}^{4} f(x)\,dx$?

A. 5

B. 8

C. 2

D. 10

E. -5

Question 10

Suppose that $\int_{-3}^{4} f(x)\,dx = 3$, $\int_{-1}^{7} f(x)\,dx = 7$, and $\int_{-3}^{7} f(x)\,dx = 5$. What is $\int_{-1}^{4} f(x)\,dx$?

A. 5

B. 8

C. 2

D. 10

E. -5

Question 11

Find the value of $\sum_{n=0}^{\infty} \frac{\sin(\frac{n\pi}{2})}{n!}$.

A. $\cos 1$

B. $\ln 2$

C. $\sin 1$

D. 1

E. e

Question 11

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A. $\cos 1$

B. $\ln 2$

C. $\sin 1$

D. 1

E. e

Question 12

If $x^2 + y^2 = 1$, then what is $\dfrac{d^2y}{dx^2}$?

A. $\dfrac{y^3}{x^2 - y^2}$

B. $-\dfrac{1}{y^3}$

C. $-\dfrac{x}{y}$

D. $-\dfrac{3x}{y^5}$

E. $\dfrac{y^2 - x^2}{y^3}$

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C. $-\dfrac{x}{y}$

D. $-\dfrac{3x}{y^5}$

E. $\dfrac{y^2 - x^2}{y^3}$

Question 13

Which of the following is not a correct solution to the problem $\int 2 \tan x \sec^2 x \, dx$?

A. $\dfrac{2}{1+\cos 2x}$

B. $\sec^2 x$

C. $\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

D. $\dfrac{4}{(e^{ix}+e^{-ix})^2}$

E. $\dfrac{1-\cos 2x}{1+\cos 2x}$

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B. $\sec^2 x$

C. $\sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$

D. $\dfrac{4}{(e^{ix} + e^{-ix})^2}$

E. $\dfrac{1 - \cos 2x}{1 + \cos 2x}$

Question 14

Rolle's theorem is a special case of which of the following theorems?

A. Extreme value theorem

B. Mean value theorem

C. Fundamental theorem of calculus

D. Intermediate value theorem

E. Mean value theorem for integrals

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Question 15

Suppose that the position of a particle is given by the equation $x = \sin(\pi t)$. Find its velocity when $t = 1/6$, given that x is time and t is position.

A. $\dfrac{\sqrt{3}\pi}{2}$

B. $\dfrac{2}{\sqrt{3}\pi}$

C. $\dfrac{4}{3\sqrt{3}\pi}$

D. $\dfrac{6}{\sqrt{35}\pi}$

E. 0

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D. $\dfrac{6}{\sqrt{35}\pi}$

E. 0

Question 16

What is the behavior of the expression $\lim_{m\to\infty}\lim_{n\to\infty}\cos^{2n}(m!\pi x)$? Assume m and n are integers.

A. It is always zero

B. It is always one

C. It does not exist

D. It is zero if x is irrational and one if x is rational

E. It is zero if x is not an integer and one if x is an integer

Question 16

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D. It is zero if x is irrational and one if x is rational

E. It is zero if x is not an integer and one if x is an integer

Question 17

Identify the differential equations whose solutions would be underestimated by Euler's method.

I. $\frac{dx}{dt} = 1$

II. $\frac{dx}{dt} = t$

III. $\frac{dx}{dt} = x$

A. II only

B. III only

C. I and II only

D. II and III only

E. I, II, and III

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B. III only

C. I and II only

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E. I, II, and III

Question 18

Find $\displaystyle\int_{\pi}^{\pi^2} \frac{\pi^2 \ln(\pi^\pi)}{\theta(\pi+\pi^2)}\, d\theta.$

A. $\dfrac{2\pi(\ln \pi)^2}{1+\pi}$

B. $\dfrac{\pi^2 \ln \pi}{1+\pi}$

C. $\dfrac{2\pi^2(\ln \pi)^2}{1+\pi}$

D. $\dfrac{\pi^2(\ln \pi)^2}{1+\pi}$

E. $\dfrac{\pi^3(\pi-1)\ln \pi}{\pi+1}$

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A. $\dfrac{2\pi(\ln\pi)^2}{1+\pi}$

B. $\dfrac{\pi^2 \ln\pi}{1+\pi}$

C. $\dfrac{2\pi^2(\ln\pi)^2}{1+\pi}$

D. $\dfrac{\pi^2(\ln\pi)^2}{1+\pi}$

E. $\dfrac{\pi^3(\pi-1)\ln\pi}{\pi+1}$

Question 19

Determine the surface area of the solid generated by revolving the curve $y = \sqrt{4 - x^2}$ about the x-axis.

A. 2π

B. 4π

C. $\frac{16}{3}\pi$

D. 8π

E. 16π

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Question 20

Which of the following integrals, when evaluated, gives the area of one of the regions bounded by the curves $y = \cos x$ and $y = \sin x$?

A. $\displaystyle\int_{\pi/4}^{5\pi/4} \cos x - \sin x \, dx$

B. $\displaystyle\int_{0}^{\pi/4} \cos x - \sin x \, dx$

C. $\displaystyle\int_{\pi/4}^{-3\pi/4} \sin x - \cos x \, dx$

D. $\displaystyle\int_{0}^{\pi} |\cos x| - |\sin x| \, dx$

E. $\displaystyle\int_{0}^{\pi} \cos x - \sin x \, dx$

Question 20

Which of the following integrals, when evaluated, gives the area of one of the regions bounded by the curves $y = \cos x$ and $y = \sin x$?

A. $\displaystyle\int_{\pi/4}^{5\pi/4} \cos x - \sin x \, dx$

B. $\displaystyle\int_{0}^{\pi/4} \cos x - \sin x \, dx$

C. $\displaystyle\int_{\pi/4}^{-3\pi/4} \sin x - \cos x \, dx$

D. $\displaystyle\int_{0}^{\pi} |\cos x| - |\sin x| \, dx$

E. $\displaystyle\int_{0}^{\pi} \cos x - \sin x \, dx$

Question 21

What is the kth derivative of x^n?

A. $k!x^{n-k}$

B. $\dfrac{n!}{(n-k-1)!}x^{n-k}$

C. $\dfrac{n!}{(n-k)!}x^{n-k}$

D. $n!x^{n-k}$

E. $(n-k)!x^{n-k}$

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B. $\dfrac{n!}{(n-k-1)!}x^{n-k}$

C. $\dfrac{n!}{(n-k)!}x^{n-k}$

D. $n!x^{n-k}$

E. $(n-k)!x^{n-k}$

Question 22

What is the average value of the function $f(x) = \sqrt{1 - x^2}$ over its domain?

A. $\pi/4$

B. $\pi/6$

C. $3/4$

D. $\sqrt{3}/2$

E. $5/8$

Question 22

What is the average value of the function $f(x) = \sqrt{1 - x^2}$ over its domain?

A. $\pi/4$

B. $\pi/6$

C. $3/4$

D. $\sqrt{3}/2$

E. $5/8$

Question 23

Evaluate $\lim_{x\to\infty} \dfrac{\sum_{n=3}^{9} nx^n}{\sqrt[3]{\sum_{n=9}^{27} \frac{x^n}{n}}}$.

A. 0

B. 3

C. 9

D. 27

E. ∞

Question 23

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A. 0

B. 3

C. 9

D. 27

E. ∞

Question 24

Evaluate $\dfrac{d}{dx}\left[\ln\ln\ln\ln x\right]$ at $x = e^{(e^e)}$.

A. e^{-e-1}

B. e^{e^e+e+1}

C. e^{-e^e-e-1}

D. $e^{-e^{-e-1}-e}$

E. e^{-e^2-e-1}

Question 24

Evaluate $\dfrac{d}{dx}\left[\ln\ln\ln\ln x\right]$ at $x = e^{(e^e)}$.

A. e^{-e-1}

B. e^{e^e+e+1}

C. e^{-e^e-e-1}

D. $e^{-e^{-e-1}-e}$

E. e^{-e^2-e-1}

Question 25

Find the value of $\lim_{x \to 0} \dfrac{\ln(1-x) - \sin x}{1 - \cos^2 x}$.

A. The limit does not exist!

B. ∞

C. $1/2$

D. $-\infty$

E. 0

Question 25

Find the value of $\lim_{x \to 0} \dfrac{\ln(1-x) - \sin x}{1 - \cos^2 x}$.

A. The limit does not exist!

B. ∞

C. $1/2$

D. $-\infty$

E. 0

Question 26

Taken collectively, how many different intervals of convergence do the following series have?

$$\sum_{n=1}^{\infty} r^n \qquad \sum_{n=1}^{\infty} n^{-r} \qquad \sum_{n=1}^{\infty} r^{-n}$$

$$\sum_{n=1}^{\infty} n^r \qquad \sum_{n=1}^{\infty} (-r)^n \qquad \sum_{n=1}^{\infty} (-r)^{-n}$$

A. 1

B. 2

C. 3

D. 4

E. 6

Question 26

Taken collectively, how many different intervals of convergence do the following series have?

$$\sum_{n=1}^{\infty} r^n \qquad \sum_{n=1}^{\infty} n^{-r} \qquad \sum_{n=1}^{\infty} r^{-n}$$

$$\sum_{n=1}^{\infty} n^r \qquad \sum_{n=1}^{\infty} (-r)^n \qquad \sum_{n=1}^{\infty} (-r)^{-n}$$

A. 1

B. 2

C. 3

D. 4

E. 6

Question 27

How many points of inflection can a polynomial of degree 7 have, at most?

A. 3

B. 5

C. 6

D. 7

E. 8

Question 27

How many points of inflection can a polynomial of degree 7 have, at most?

A. 3

B. 5

C. 6

D. 7

E. 8

Question 28

Evaluate $\dfrac{d}{dx}\left[(\ln x)^{\ln x}\right]$.

A. $(\ln x)^{\ln x}\left(\dfrac{\ln x + \ln\ln x}{x}\right)$

B. $(\ln x)^{\ln x}\left(\dfrac{1 + \ln\ln x}{x}\right)$

C. $(\ln x)^{\ln x}\left(\dfrac{1 + \ln x}{\ln\ln x}\right)$

D. $(\ln x)^{\ln x}\left(1 + \dfrac{\ln\ln x}{x}\right)$

E. $(\ln x)^{\ln x}\left(\dfrac{1 + \ln\ln x}{\ln x}\right)$

Question 28

Evaluate $\dfrac{d}{dx}\left[(\ln x)^{\ln x}\right]$.

A. $(\ln x)^{\ln x}\left(\dfrac{\ln x + \ln\ln x}{x}\right)$

B. $(\ln x)^{\ln x}\left(\dfrac{1 + \ln\ln x}{x}\right)$

C. $(\ln x)^{\ln x}\left(\dfrac{1 + \ln x}{\ln\ln x}\right)$

D. $(\ln x)^{\ln x}\left(1 + \dfrac{\ln\ln x}{x}\right)$

E. $(\ln x)^{\ln x}\left(\dfrac{1 + \ln\ln x}{\ln x}\right)$

Question 29

Evaluate $\int_{-\infty}^{0} x^5 e^x \, dx$.

A. -120

B. -24

C. -5

D. -1

E. The integral cannot be expressed in closed form

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A. -120

B. -24

C. -5

D. -1

E. The integral cannot be expressed in closed form

Question 30

The power rule states that $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$. In which of the following situations does the power rule not hold?

I. $x = 0$ and $n < 1$

II. $x = 0$ and $n = 1$

III. $x < 0$ and n is irrational

A. None

B. I only

C. II only

D. I and II only

E. I, II, and III

Question 30

The power rule states that $\frac{d}{dx}\left[x^n\right] = nx^{n-1}$. In which of the following situations does the power rule not hold?

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II. $x = 0$ and $n = 1$

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B. I only

C. II only

D. I and II only

E. I, II, and III

Question 31

Compute $\int_0^\infty (e^{-x})^2\,dx$.

A. $1/2$

B. $\sqrt{\pi}/2$

C. 1

D. $\sqrt{\pi}$

E. The integral does not converge

Question 31

Compute $\int_0^\infty (e^{-x})^2 \, dx$.

A. $1/2$

B. $\sqrt{\pi}/2$

C. 1

D. $\sqrt{\pi}$

E. The integral does not converge

Question 32

If the arc length of the curve given by $x(t) = \cos\sqrt{t}$ and $y(t) = \sin\sqrt{t}$ from $t = 0$ to $t = T$ is 6π, then what is T?

A. $\sqrt{3}$

B. $\sqrt{6}$

C. 3

D. 6

E. 9

Question 32

If the arc length of the curve given by $x(t) = \cos\sqrt{t}$ and $y(t) = \sin\sqrt{t}$ from $t = 0$ to $t = T$ is 6π, then what is T?

A. $\sqrt{3}$

B. $\sqrt{6}$

C. 3

D. 6

E. 9

Question 33

Evaluate $\frac{d}{dx}\left[x + x^2 + x^3 + \cdots + x^{100}\right]$ at $x = 1$.

A. 99

B. 100

C. 5050

D. 10100

E. 100!

Question 33

Evaluate $\frac{d}{dx}\left[x + x^2 + x^3 + \cdots + x^{100}\right]$ at $x = 1$.

A. 99

B. 100

C. 5050

D. 10100

E. 100!

Question 34

What is the rate of change of the length of the hypotenuse of an isosceles right triangle with respect to the length of one of its legs?

A. 0

B. 1

C. $\sqrt{2}$

D. 2

E. $2\sqrt{2}$

Question 34

What is the rate of change of the length of the hypotenuse of an isosceles right triangle with respect to the length of one of its legs?

A. 0
B. 1
C. $\sqrt{2}$
D. 2
E. $2\sqrt{2}$

Question 35

What is $\frac{d}{dA}\left[ABRACADABRA\right]$ when $A = 1$, $B = 2$, $C = 3$, and $D = 4$?

A. $5R^2$

B. $48R^2$

C. $120R^2$

D. $192R^2$

E. $240R^2$

Question 35

What is $\frac{d}{dA}\left[ABRACADABRA\right]$ when $A = 1$, $B = 2$, $C = 3$, and $D = 4$?

A. $5R^2$

B. $48R^2$

C. $120R^2$

D. $192R^2$

E. $240R^2$

Question 36

Determine the value of $\frac{d}{dx}\left[|\sin x| + |\cos x| + |\tan x|\right]$ at $x = \frac{\pi}{4}$.

A. 1

B. $\sqrt{2}$

C. $1 + \frac{\sqrt{2}}{2}$

D. 2

E. $1 + \sqrt{2}$

Question 36

Determine the value of $\frac{d}{dx}\left[|\sin x| + |\cos x| + |\tan x|\right]$ at $x = \frac{\pi}{4}$.

A. 1

B. $\sqrt{2}$

C. $1 + \frac{\sqrt{2}}{2}$

D. 2

E. $1 + \sqrt{2}$

Question 37

Compute $\dfrac{d}{dx}\left[\dfrac{x}{\ln x}\right]$.

A. $\dfrac{1}{\ln x} + \dfrac{1}{(\ln x)^2}$

B. $\dfrac{1}{\ln x} - \dfrac{1}{(\ln x)^2}$

C. $\dfrac{1}{x^2} - \dfrac{\ln x}{x^2}$

D. $\dfrac{1}{(\ln x)^2} - \dfrac{1}{\ln x}$

E. $\dfrac{\ln x - 1}{\ln(x^2)}$

Question 37

Compute $\dfrac{d}{dx}\left[\dfrac{x}{\ln x}\right]$.

A. $\dfrac{1}{\ln x} + \dfrac{1}{(\ln x)^2}$

B. $\dfrac{1}{\ln x} - \dfrac{1}{(\ln x)^2}$

C. $\dfrac{1}{x^2} - \dfrac{\ln x}{x^2}$

D. $\dfrac{1}{(\ln x)^2} - \dfrac{1}{\ln x}$

E. $\dfrac{\ln x - 1}{\ln(x^2)}$

Question 38

Suppose that the radius of a sphere is 1 inch, and its volume is increasing at a rate of 1 cubic inch per minute. How fast is its surface area increasing?

A. $1\ \text{in}^2/\text{min}$

B. $2\ \text{in}^2/\text{min}$

C. $2\pi\ \text{in}^2/\text{min}$

D. $4\pi\ \text{in}^2/\text{min}$

E. $8\pi\ \text{in}^2/\text{min}$

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Suppose that the radius of a sphere is 1 inch, and its volume is increasing at a rate of 1 cubic inch per minute. How fast is its surface area increasing?

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C. $2\pi\ \text{in}^2/\text{min}$

D. $4\pi\ \text{in}^2/\text{min}$

E. $8\pi\ \text{in}^2/\text{min}$

Question 39

What is the average value of $\arctan x$ from $x = 0$ to $x = 1$?

A. $\frac{\pi}{4} - \frac{1}{2}$

B. $\frac{\pi}{4} - \ln \sqrt{2}$

C. $\frac{1}{4} + \ln 2$

D. $\frac{\pi}{2} - \frac{3}{4}$

E. $\ln\left(\frac{\pi}{2}\right)$

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B. $\frac{\pi}{4} - \ln \sqrt{2}$

C. $\frac{1}{4} + \ln 2$

D. $\frac{\pi}{2} - \frac{3}{4}$

E. $\ln\left(\frac{\pi}{2}\right)$

Question 40

Find the average value of $\tan x$ for x in the interval $[0, \pi/4]$.

A. $\frac{1}{2}$

B. $\frac{2}{\pi} + \frac{1}{4}$

C. $1 - \frac{2}{\pi}$

D. $\frac{\pi}{4}$

E. $\frac{4}{\pi} - 1$

Question 40

Find the average value of $\tan x$ for x in the interval $[0, \pi/4]$.

A. $\frac{1}{2}$

B. $\frac{2}{\pi} + \frac{1}{4}$

C. $1 - \frac{2}{\pi}$

D. $\frac{\pi}{4}$

E. $\frac{4}{\pi} - 1$

Question 41

Find $\lim_{x \to 0} |x|^x$.

A. 0

B. $1/2$

C. 1

D. ∞

E. The limit does not exist

Question 41

Find $\lim_{x \to 0} |x|^x$.

A. 0

B. 1/2

C. 1

D. ∞

E. The limit does not exist

Question 42

Evaluate $\dfrac{d}{dx}\displaystyle\int_x^{2x} \frac{x}{t}\,dt$.

A. $2x$

B. $\ln x$

C. $\ln 2$

D. 1

E. x

Question 42

Evaluate $\dfrac{d}{dx}\displaystyle\int_x^{2x} \frac{x}{t}\,dt$.

A. $2x$

B. $\ln x$

C. $\ln 2$

D. 1

E. x

Question 43

Find $\int_{-1}^{2} \frac{dx}{x}$.

A. 0

B. $\ln 2$

C. 1

D. e

E. The integral does not converge

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A. 0

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Question 44

For what value of p is the expression

$$\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{n^p}$$ finite and nonzero?

A. $1/2$

B. 1

C. $3/2$

D. 2

E. $5/2$

Question 44

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$$\lim_{n\to\infty} \frac{\sqrt{1}+\sqrt{2}+\sqrt{3}+\cdots+\sqrt{n}}{n^p}$$ finite and nonzero?

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B. 1

C. 3/2

D. 2

E. 5/2

Question 45

Suppose that $x^y = y^x$. Find $\dfrac{dy}{dx}$.

A. $\dfrac{x^2 \ln y - xy}{y^2 \ln x - xy}$

B. $\dfrac{xy \ln y - y^2}{xy \ln x - x^2}$

C. $\dfrac{xy \ln y - x^2}{xy \ln x - y^2}$

D. $\dfrac{x^2 \ln x - y^2}{y^2 \ln y - x^2}$

E. $\dfrac{xy \ln x - y^2}{xy \ln y - x^2}$

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D. $\dfrac{x^2 \ln x - y^2}{y^2 \ln y - x^2}$

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Question 46

Consider a collection of spheres with radii given by $\left\{1, \frac{1}{2^p}, \frac{1}{3^p}, \ldots\right\}$. If the total volume is finite but the total surface area is infinite, then what values of p are possible?

A. $2 \leq p < 3$

B. $\frac{1}{3} < p \leq \frac{1}{2}$

C. $2 < p \leq 3$

D. $\frac{1}{3} < p \leq 1$

E. $1 < p \leq 3$

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E. $1 < p \leq 3$

Question 47

Determine the value of $\sqrt{1+\sqrt{1+\sqrt{1+\cdots}}}$.

A. $\frac{\sqrt{2}}{5}$

B. $\frac{1-\sqrt{5}}{2}$

C. $\frac{1-\sqrt{3}}{2}$

D. $\frac{1+\sqrt{5}}{2}$

E. $\frac{1+\sqrt{3}}{2}$

Question 47

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C. $\frac{1-\sqrt{3}}{2}$

D. $\frac{1+\sqrt{5}}{2}$

E. $\frac{1+\sqrt{3}}{2}$

Question 48

Determine $\dfrac{d}{dx}\left[(\ln x)^e\right]$.

A. $\dfrac{e}{x}(\ln x)^{e-1}$

B. $(\ln \ln x)(\ln x)^e$

C. $\dfrac{e}{x}$

D. $(\ln x)^e \left(1 + \dfrac{e}{x}\right)$

E. 1

Question 48

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B. $(\ln \ln x)(\ln x)^e$

C. $\dfrac{e}{x}$

D. $(\ln x)^e \left(1 + \dfrac{e}{x}\right)$

E. 1

Question 49

Suppose that f is a function defined on an interval I. Which of the following statements must be true?

I. If f is continuous on I, then f is differentiable on I.

II. If f is differentiable on I, then f is continuous on I.

III. If f is integrable on I, then f is continuous on I.

IV. If f is continuous on I, then f is integrable on I.

A. II only

B. I and III only

C. II and IV only

D. II, III, and IV only

E. I, II, III, and IV

Question 49

Suppose that f is a function defined on an interval I. Which of the following statements must be true?

I. If f is continuous on I, then f is differentiable on I.

II. If f is differentiable on I, then f is continuous on I.

III. If f is integrable on I, then f is continuous on I.

IV. If f is continuous on I, then f is integrable on I.

A. II only

B. I and III only

C. II and IV only

D. II, III, and IV only

E. I, II, III, and IV